

Write the complete solution

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \rightarrow \begin{aligned} u + 2v + 2w &= 1 \\ 2u + 2v + 4w &= 4 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix} u + \begin{bmatrix} 2 \\ 4 \end{bmatrix} v + \begin{bmatrix} 2 \\ 4 \end{bmatrix} w =$$

$Ax = b$

Find the value of c that makes it possible to solve $Ax = b$, and solve it:

$$x_p = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{aligned} u + v + 2w &= 2 \\ 2u + 3v - w &= 5 \\ 3u + 4v + w &= c \end{aligned}$$

$$\begin{bmatrix} -b \\ 6 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 3 & -1 & 5 \\ 3 & 4 & 1 & c \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -1 & -5 & -1 \\ 0 & -1 & 6 & c \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -1 & -5 & -1 \\ 0 & 0 & 0 & c-7 \end{bmatrix}$$

$$\begin{aligned} -v + 5w &= -1 \\ v &= +5w \\ u + v + 2w &= 2 \\ u + 1 + 5w + 2w &= 2 \\ u &= 1 - 7w \end{aligned}$$

$$\begin{bmatrix} -7 \\ 5 \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 5 \\ 1 \end{bmatrix} a$$